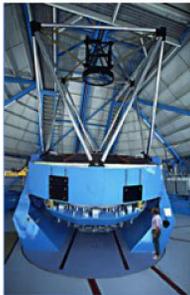


# The Sun Is Like A Watermelon

Comparison Of Stellar Structure Models to Observational Data

Benjamin Thompson



## Why a Watermelon?

If you go to the supermarket to purchase a watermelon, how do you tell it is ripe?



You knock on it! If the watermelon sounds hollow, then it is ripe. If it sounds solid, then it still needs more time.



When considering buying a watermelon, you want to know the internal characteristics (i.e., whether it is ripe or not), but you are only able to observe the outside. Stars pose a similar problem.

Unfortunately, we want to know more about a star than whether it is ripe or not. This poster will walk through the theory of stellar structure, then illustrate how models compare to observational data.

## [1] Basic Equations of Stellar Structure

Stellar structure models are built using 5 basic equations:

### Equilibrium

$$\frac{dP}{dr} = - \left( \frac{GM_r}{r^2} \right) \rho$$

We know that stars are stable, as they live for billions of years without collapsing. Gravity constantly pulls the outer layers of the star towards the center, so a pressure must be exerted outwards to counteract it. That is what the equation describes. The quantity in parentheses is the acceleration due to gravity at a specific radius.

### Mass Continuity

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

This equation is simple: the amount of mass enclosed within a radius  $r$  increases as  $r$  increases.

### Energy Generation

$$\frac{dL}{dr} = (4\pi r^2 \rho) \epsilon$$

This equation describes how energy is generated within the star. The quantity in parentheses is the amount of mass within a spherical shell at radius  $r$ , while  $\epsilon$  is the amount of energy generated per kilogram.  $\epsilon$  will change based on what fusion process is happening at different layers of the star.

### Energy Transport

Energy generation via fusion in stars happens near the core, yet photons emerge from the star's surface. The energy generated near the core is transported to the surface via two different methods: radiation or convection.

Energy transport will occur via whichever method is most efficient: if the gas is highly opaque, convection will be favored.

$$\frac{dT_{rad}}{dr} = - \frac{3}{16\sigma} \frac{\kappa \rho}{T^3} \left( \frac{L}{4\pi r^2} \right) \quad \text{Radiation}$$

Here,  $\sigma$  is the Stefan-Boltzmann constant, and  $\kappa$  is the mean opacity.

$$\frac{dT_{ad}}{dr} = - \left( 1 - \frac{C_V}{C_P} \right) \frac{\mu m_H}{k} \frac{GM_r}{r^2} \quad \text{Convection}$$

Here,  $C_V$  and  $C_P$  are specific heats for constant volume and pressure, and  $\mu$  is the average molecular weight. Convection is assumed to happen adiabatically: no heat is lost to the outside during the process.

## [2] Additional Equations

Above we have 5 equations of stellar structure, but 8 unknowns:  $P$ ,  $M$ ,  $L$ ,  $T$ ,  $\kappa$ ,  $\rho$ ,  $\epsilon$ . We must find 3 additional equations.

### Equation of State (EOS)

$$P = \frac{\rho k T}{\mu m_H} + \frac{4}{3c} \sigma T^4$$

This relates density and pressure. The first term comes from the ideal gas law, and the second is from radiation pressure.

### Opacity and Energy Generation

Mean opacity,  $\kappa$ , and energy generation per kilogram,  $\epsilon$ , are generated from other models, and are taken as givens. They are functions of  $\rho$ ,  $T$ , and  $X_i$ , the number densities of various elements (H, He, Li, etc.) within the layer of the star.

With these three other equations, the system is solvable.

## [3] Solutions

Except for very specific opacity and energy generation models, the stellar structure system of equations does not have an analytic solution. Instead, it must be numerically integrated through the following steps. For each step, we address conditions within a layer of the star at radius  $r$  from the center:

1. Define  $X_i$  for the layer (this composition depends on the evolutionary state of the star we are interested in, i.e. whether much of the layer has been converted into something besides H + He).
2. Determine  $\kappa$  and  $\epsilon$  for the layer using  $X_i$  and  $\rho$  and  $T$  from the previous layer.
3. Determine which energy transport method is most efficient (you can do this from the determined  $\kappa$ ).
4. Use the four fundamental equations above to determine  $P$ ,  $M$ ,  $L$ ,  $T$  for the layer.

This is usually done via an iterative process, where surface boundary conditions (total mass, luminosity and radius) are chosen, and then integrated to see if core boundary conditions are met ( $M$ ,  $L$  = 0). If this is not the case, we change one of the outside parameters ( $M$ ,  $L$ ,  $R$ ) and try again.

## [4] A Spherical Cow in a Vacuum

Physicists' love of ideal cases is well known, and is apparent in the basic equations stated above. What non-ideal effects are we glossing over?

**General Relativity:** The Newtonian gravity model that is used above (see Equilibrium) begins to break down at stellar masses. To be more accurate, we must use general relativity formulas for gravity in our equations. While messier to write, integrating this into our models is not difficult.

**Coulomb Interaction:** The above models assume that the star is an ideal gas. We know this is not the case for gases at the high temperatures and pressures inside many stars. We must take into account the electromagnetic forces between ions within the star.

**Degeneracy Pressure:** Only one fermion (electron, proton, neutron) may occupy a particular quantum state at a time. With high densities inside stars, this manifests as an outward pressure, adding to the photon pressure already present.

**Special Relativity:** At certain temperature and pressures, the ionized electrons within the star begin moving at appreciable fractions of the speed of light, necessitating special relativity considerations.

## [5] Testing the Models

There are several published stellar structure models in wide use today. To the right, we show four of the most popular models, and compare them to observational data of the star cluster M67 (shown in grey).

Equations that determine opacity ( $\kappa$  above), are usually written separately, and there are several opacity codes that are widely used. The opacity code used is noted along with what non-ideal effects are included in the equation of state.

Many stellar structure models use equations that are only valid for certain masses of stars. We break up the underlying physics into high and low mass ranges to the right. The transition occurs near the dashed black lines.

The plots to the right are Color-Magnitude Diagrams (CMDs), which are observational versions of the Hertzsprung-Russel Diagram. The vertical axis decreases with increasing star brightness, while the bottom axis increases with decreasing temperature (hotter stars are on the left).

### PARSEC

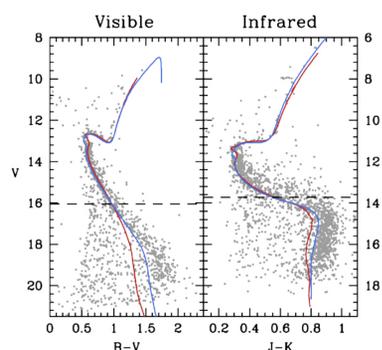
(Bressan et al. 2012)

#### High Mass Parameters

Opacity Code: OPAL  
EOS: "FreeEOS" (CI+DP+SR)

#### Low Mass Parameters

Opacity Code: AESOPUS  
EOS: "FreeEOS" (CI+DP+SR)



### Dartmouth

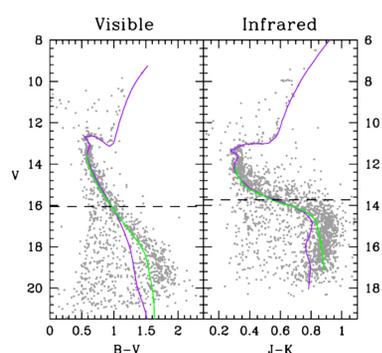
(Dotter et al. 2007)

#### High Mass Parameters

Opacity Code: OPAL  
EOS: Ideal Gas + CI

#### Low Mass Parameters

Opacity Code: A&F  
EOS: "FreeEOS" (CI+DP+SR)



### Padova

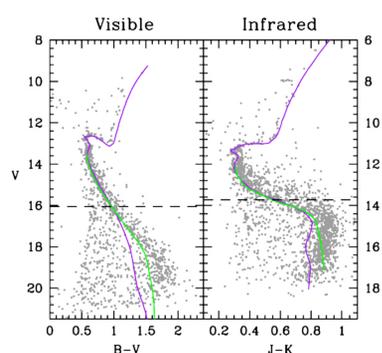
(Girardi et al. 2002)

#### High Mass Parameters

Opacity Code: OPAL  
EOS: Ideal Gas + DP + CI

#### Low Mass Parameters

Opacity Code: A&F  
EOS: "MHD" (CI+DP)



### YREC

(Deokkeun et al. 2006)

#### High Mass Parameters

Opacity Code: OPAL  
EOS: OPAL (CI)

#### Low Mass Parameters

Opacity Code: A&F  
EOS: "SCVH" (CI+DP+SR)

(Empirically Calibrated)

## [6] Conclusions

There are several important things illustrated by the comparison plots:

1. Despite differences in EOS physics, all the models work extremely well on high-mass stars. Since the opacity codes are all the same, this means that degeneracy pressure and special relativity effects are very weak (or nonexistent) for these stars. This makes sense since high-mass stars are, generally, not very dense.
2. Most models do not work well for very low-mass stars. The PARSEC and Padova models vary greatly, while Dartmouth varies much less. The YREC system seems to work well, but it is hard to determine why. The YREC system has empirically calibrated CMDs, so it is difficult to determine whether the goodness-of-fit is due to the underlying model physics, or simply the translation from stellar parameters to observables shown in the CMD.
3. The AESOPUS opacity code is very similar to that of A&F. This means that, for low mass, PARSEC and DARTMOUTH have very similar physical equations, yet they are widely discrepant on the CMDs. This deviation can be from one of two effects: the  $X_i$  values are different between the two models, and may cause the same equations to give different results, or the atmosphere model (going from stellar parameters to observable properties) of PARSEC gives incorrect values.

We plan to test these models on many more clusters in order to help solve some of these issues.

WOCS

WIYN Open Cluster Study